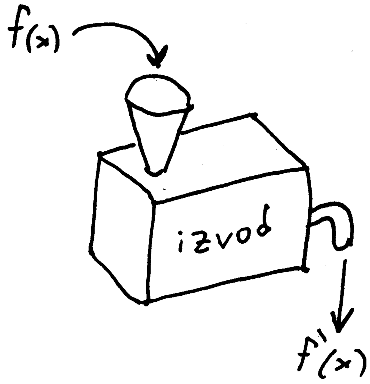
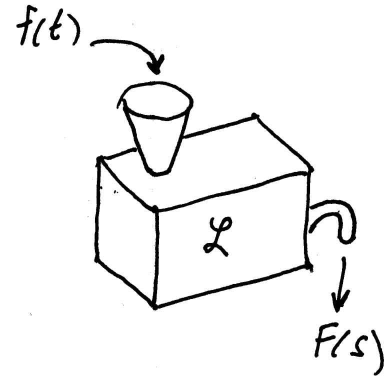
Definicija Laplasove transformacije





Neka je $f(t)$ f-ja na $[0,\infty)$. Laplasova transformacija od f je f-ja F definisana pomoću integrala

$$F(s) := \int_0^\infty e^{-st} f(t)\, dt. \qquad \dots (1)$$

Domen f-je $F(s)$ su sve vrijednosti od s za koje integral (1) postoji. Laplasovu transformaciju f-je f ćemo označavati ili sa F ili sa $\mathcal{L}\{f\}$.

Linearnost Laplasove transformacije:
Ako je $\mathcal{L}\{f\} = F$ i $\mathcal{L}\{g\} = G$ tada vrijedi

$$\underline{\mathcal{L}\{a\, f(t) + b\, g(t)\}(s) = aF(s) + b\, G(s)}$$

gdje su a i b konstante.

F-ja $f(t)$ se naziva orginal, a f-ja $F(s) = \mathcal{L}\{f(t)\}(s)$ slika.

(#) Odrediti Laplasovu transformaciju konstantne f-je $f(t)=1$, $t \geq 0$.

Rj.

Laplasova transformacija f-je f je f-ja F definisana pomoću integrala

$$F(s) := \int_0^\infty e^{-st} f(t)\,dt.$$

Laplasova transformacija f-je f označavamo sa F ili sa $\mathcal{L}\{f\}$.

U našem slučaju

$$F(s) = \int_0^\infty e^{-st} \cdot 1\,dt = \lim_{N\to\infty} \int_0^N e^{-st}\,dt = \left| \begin{array}{l} d(-st) = -s\,dt \\ dt = \frac{1}{-s}\,d(-st) \end{array} \right|$$

$$= \frac{-1}{s} \lim_{N\to\infty} \int_0^N e^{-st}\,d(-st) = \frac{-1}{s} \lim_{N\to\infty} e^{-st}\Big|_0^N = \frac{-1}{s} \lim_{N\to\infty} \left(e^{-sN} - 1\right) =$$

$$= \left| \begin{array}{l} \lim\limits_{N\to\infty} e^{-sN} = \lim\limits_{N\to\infty} \frac{1}{e^{sN}} = 0 \quad \text{za } s>0 \\ \lim\limits_{N\to\infty} e^{-sN} = \infty \quad \text{za } s<0 \end{array} \right| = \begin{cases} -\frac{1}{s}(0-1) = \frac{1}{s}, & \text{za } s>0 \\ -\frac{1}{s} \cdot \infty = \infty, & \text{za } s<0 \end{cases}$$

Prema tome

$F(s) = \frac{1}{s}$ gdje je domen od F svi $s>0$

$$\mathcal{L}\{1\}(s) = \frac{1}{s}, \quad s>0$$

Odrediti Laplasovu transformaciju f-je $f(t)=e^{\alpha t}$, gdje je α konstanta.

Rj. Laplasova transformacija od f je f-ja F definisana sa

$$F(s):=\int_0^\infty e^{-st}f(t)\,dt$$

Laplasovu transformaciju f-je f označavamo sa F ili $\mathcal{L}\{f\}$.

U ovom slučaju

$$F(s)=\int_0^\infty e^{-st}\cdot e^{\alpha t}\,dt=\int_0^\infty e^{(\alpha-s)t}\,dt=\left|\begin{array}{c} d((\alpha-s)t)=(\alpha-s)dt \\ dt=\frac{1}{\alpha-s}\,d((\alpha-s)t)\end{array}\right|=$$

$$=\frac{1}{\alpha-s}\lim_{N\to\infty}\int_0^N e^{(\alpha-s)t}\,d((\alpha-s)t)=\frac{1}{\alpha-s}\lim_{N\to\infty}e^{(\alpha-s)t}\Big|_0^N=$$

$$=\frac{1}{\alpha-s}\lim_{N\to\infty}\left(e^{(\alpha-s)N}-1\right)=\left|\begin{array}{l}\lim\limits_{N\to\infty}e^{(\alpha-s)N}=0,\ \text{za } \alpha-s<0 \\ \lim\limits_{N\to\infty}e^{(\alpha-s)N}=\infty,\ \text{za } \alpha-s>0\end{array}\right|$$

$$=\begin{cases}\frac{1}{\alpha-s}(0-1)=\frac{1}{s-\alpha}, & \text{za } \alpha<s \\ \frac{1}{\alpha-s}(\infty-1)=\infty, & \text{za } \alpha>s\end{cases}$$

Prema tome $F(s)=\frac{1}{s-\alpha}$ gdje je domen od F svi $s>\alpha$.

$$\mathcal{L}\{e^{\alpha t}\}(s)=\frac{1}{s-\alpha},\quad s>\alpha.$$

(#) Odrediti $\mathcal{L}\{\sin \beta t\}$ gdje je β nenula konstanta.

Rj. Laplasova transformacija od f je f-ja F definisana sa

$$F(s) := \int_0^\infty e^{-st} f(t)\, dt$$

Laplasovu transformaciju f-je f označavamo sa F ili sa $\mathcal{L}\{f\}$.

$$F(s) = \int_0^\infty e^{-st} \sin \beta t\, dt = \lim_{N\to\infty} \int_0^N e^{-st} \sin \beta t\, dt$$

Izračunajmo posebno integral $I = \int e^{-st} \sin \beta t\, dt$.

$$I = \left| \begin{array}{ll} u = \sin \beta t & dv = e^{-st} dt \\ du = \beta \cos \beta t\, dt & v = \frac{-1}{s} e^{-st} \end{array} \right| = \frac{-1}{s} e^{-st} \sin \beta t + \frac{\beta}{s} \int e^{-st} \cos \beta t\, dt$$

$$= \left| \begin{array}{ll} u = \cos \beta t & dv = e^{-st} dt \\ du = \beta(-\sin \beta t) & v = \frac{-1}{s} e^{-st} \end{array} \right| = \frac{-1}{s} e^{-st} \sin \beta t + \frac{\beta}{s} \Big[-\frac{1}{s} e^{-st} \cos \beta t$$

$$- \frac{\beta}{s} \int e^{-st} \sin \beta t \Big] = -\frac{1}{s} e^{-st} \sin \beta t - \frac{\beta}{s^2} e^{-st} \cos \beta t - \frac{\beta^2}{s^2} \cdot I$$

$$\Rightarrow \quad I + \frac{\beta^2}{s^2} I = e^{-st} \left(-\frac{1}{s} \sin \beta t - \frac{\beta}{s^2} \cos \beta t \right)$$

$$\frac{s^2 + \beta^2}{s^2} I = e^{-st} \left(-\frac{1}{s} \sin \beta t - \frac{\beta}{s^2} \cos \beta t \right) \quad / \cdot \frac{s^2}{s^2 + \beta^2}$$

$$I = \int e^{-st} \sin \beta t\, dt = \frac{e^{-st}}{s^2 + \beta^2} \left(-s \sin \beta t - \beta \cos \beta t \right)$$

Prema tome

$$F(s) = \lim_{N\to\infty} \left(\frac{e^{-st}}{s^2+\beta^2} (-s \sin\beta t - \beta\cos\beta t) \right) \Bigg|_0^N =$$

$$= \lim_{N\to\infty} \left[\frac{e^{-sN}}{s^2+\beta^2} (-s \cdot \sin\beta N - \beta \cos\beta N) - \frac{1}{s^2+\beta^2} (0 - \beta \cdot 1) \right]$$

$$= \frac{\beta}{s^2+\beta^2} \quad \text{za} \quad s > 0$$

(nije teško vidjeti da je $\lim_{N\to\infty} \frac{-s}{s^2+\beta^2} e^{-sN} \sin\beta N = 0$

kao i $\lim_{N\to\infty} \frac{-\beta}{s^2+\beta^2} e^{-sN} \cos\beta N = 0$ za $s > 0$)

Prema tome

$F(s) = \frac{\beta}{s^2+\beta^2}$ gdje je domen od F $s > 0$

$$\mathcal{L}\{\sin\beta t\}(s) = \frac{\beta}{s^2+\beta^2}, \quad s > 0$$

(#) Odrediti Laplasovu transformaciju f-je

$$f(t) = \begin{cases} 2, & 0<t<5 \\ 0, & 5<t<10 \\ e^{4t}, & 10<t \end{cases}$$

Rj.

Laplasova transformacija f-je f(t) je f-ja F(s) definisana sa

$$F(s) := \int_0^\infty e^{-st} f(t)\, dt$$

Laplasovu transformaciju f-je f(t) označavamo sa F ili sa $\mathcal{L}\{f\}$.

Kako je f(t) definisana različitim formulama na različitim intervalima, prilikom računanja integral ćemo podijeliti na dijelove

$$F(s) = \int_0^\infty e^{-st} f(t)\, dt = \int_0^5 e^{-st} \cdot 2\, dt + \int_5^{10} e^{-st} \cdot 0\, dt + \int_{10}^{\infty} e^{-st} e^{4t}\, dt =$$

$$= \left| \begin{array}{l} d(-st) = -s\,dt \\ dt = -\frac{1}{s} d(-st) \end{array} \right| = 2\int_0^5 \frac{(-1)}{s} e^{-st} d(-st) + \lim_{N\to\infty} \int_{10}^{N} e^{(4-s)t} dt =$$

$$= \frac{-2}{s} e^{-st}\Big|_0^5 + \lim_{N\to\infty} \frac{1}{4-s} e^{(4-s)t}\Big|_{10}^{N} = \frac{-2}{s}\left(e^{-5s} - 1\right) +$$

$$+ \frac{1}{4-s} \lim_{N\to\infty} \left(e^{(4-s)N} - e^{(4-s)\cdot 10}\right) = \left| \begin{array}{l} \lim\limits_{N\to\infty} e^{(4-s)N} = 0 \quad \text{za} \quad 4-s<0 \\ \lim\limits_{N\to\infty} e^{(4-s)N} = \infty \quad \text{za} \quad 4-s>0 \end{array} \right| =$$

$$= \frac{2}{s} - \frac{2}{s} e^{-5s} + \frac{(-1)}{4-s} e^{10(4-s)}, \quad \text{za} \quad s>4$$

LINEARNOST LAPLASOVE TRANSFORMACIJE

Neka su f, f_1 i f_2 f-je za koje postoji Laplasova transformacija za $s>\alpha$ i neka je c konstanta. Pokazati da za $s>\alpha$ vrijedi

$$\mathcal{L}\{f_1+f_2\} = \mathcal{L}\{f_1\} + \mathcal{L}\{f_2\}$$

$$\mathcal{L}\{cf\} = c\,\mathcal{L}\{f\}.$$

Rj. Prema definiciji $F(s)=\mathcal{L}\{f\}(s) := \int_0^\infty e^{-st} f(t)\,dt$

U ovom slučaju

$$\mathcal{L}\{f_1+f_2\}(s) = \int_0^\infty e^{-st}\big(f_1(t)+f_2(t)\big)\,dt =$$

$$= \int_0^\infty e^{-st} f_1(t)\,dt + \int_0^\infty e^{-st} f_2(t)\,dt$$

$$= \mathcal{L}\{f_1\}(s) + \mathcal{L}\{f_2\}(s)$$ što je i trebalo dobiti.

Slično

$$\mathcal{L}\{cf\}(s) = \int_0^\infty e^{-st}\big(cf(t)\big)\,dt = c\int_0^\infty e^{-st} f(t)\,dt = c\,\mathcal{L}\{f\}(s)$$

(#) Odrediti $\mathcal{L}\{11+5e^{4t}-6\sin 2t\}$.

Rj: U rješenju zadatka ćemo iskoristiti linearnost Laplasove transformacije

$$\mathcal{L}\{11+5e^{4t}-6\sin 2t\} = \mathcal{L}\{11\} + \mathcal{L}\{5e^{4t}\} + \mathcal{L}\{-6\sin 2t\}$$

$$= 11\mathcal{L}\{1\} + 5\mathcal{L}\{e^{4t}\} - 6\mathcal{L}\{\sin 2t\}.$$

U prva tri zadatka smo pokazali da je

$$\underset{s>0}{\mathcal{L}\{1\}(s) = \frac{1}{s}}, \quad \underset{s>\alpha}{\mathcal{L}\{e^{\alpha t}\}(s) = \frac{1}{s-\alpha}}, \quad \underset{s>0}{\mathcal{L}\{\sin at\}(s) = \frac{a}{s^2+a^2}}$$

Prema tome

$$\mathcal{L}\{11+5e^{4t}-6\sin 2t\}(s) = 11\cdot\frac{1}{s} + 5\cdot\frac{1}{s-4} - 6\cdot\frac{2}{s^2+2^2}$$

$$= \frac{11}{s} + \frac{5}{s-4} - \frac{12}{s^2+4}$$

Kako su svi $\mathcal{L}\{1\}$, $\mathcal{L}\{e^{4t}\}$, $\mathcal{L}\{\sin 2t\}$ definisani za $s>4$, to je i transformacija definisana za $s>4$,

Odrediti Laplasovu transformaciju f-je $f(t) = \operatorname{ch} \alpha t$ gdje je $\alpha > 0$ konstanta.

Rj: U ovom zadatku ćemo iskoristiti linearnost Laplasove transformacije

$$\mathcal{L}\{c_1 f_1 + c_2 f_2\} = c_1 \mathcal{L}\{f_1\} + c_2 \mathcal{L}\{f_2\}$$

c_1, c_2 su konstante

$$\operatorname{ch} \alpha t = \frac{e^{\alpha t} + e^{-\alpha t}}{2} = \frac{1}{2}(e^{\alpha t} + e^{-\alpha t})$$

$$\mathcal{L}\{\operatorname{ch} \alpha t\}(s) = \mathcal{L}\{\tfrac{1}{2}(e^{\alpha t} + e^{-\alpha t})\}(s) = \frac{1}{2}\mathcal{L}\{e^{\alpha t}\}(s) +$$
$$+ \frac{1}{2}\mathcal{L}\{e^{-\alpha t}\}(s) \overset{(*)}{=}$$

U jednom od prethodnih zadataka smo izračunali da je

$$\mathcal{L}\{e^{\alpha t}\}(s) = \frac{1}{s-\alpha}, \quad s > \alpha$$

Prema tome

$$\overset{(*)}{=} \frac{1}{2} \cdot \frac{1}{s-\alpha} + \frac{1}{2} \cdot \frac{1}{s+\alpha} = \frac{1}{2}\left(\frac{1}{s-\alpha} + \frac{1}{s+\alpha}\right) =$$
$$= \frac{1}{2} \cdot \frac{s+\alpha+s-\alpha}{s^2-\alpha^2} = \frac{s}{s^2-\alpha^2} \quad \text{za} \quad s > \alpha$$

Prema tome

$$\mathcal{L}\{\operatorname{ch} \alpha t\}(s) = \frac{s}{s^2-\alpha^2} \quad \text{za} \quad s > \alpha > 0$$

(#) Odrediti Laplasovu transformaciju f-je $f(t)=(t+3)^2$.

Rj: Koristit ćemo osobinu linearnosti Laplasove transformacije i elementarnu tabelu Laplasovih transformacija

$$\mathcal{L}\{(t+3)^2\}(s) = \mathcal{L}\{t^2+6t+9\}(s) =$$

$$= \mathcal{L}\{t^2\}(s)+6\mathcal{L}\{t\}(s)+9\mathcal{L}\{1\}(s) =$$

$$= \frac{2}{s^3} + 6\cdot\frac{1}{s^2} + 9\cdot\frac{1}{s}$$

$$= \frac{2}{s^3} + \frac{6}{s^2} + \frac{9}{s}$$

Odrediti Laplasovu transformaciju f-je $f(t)=\operatorname{sh} \beta t$ gdje je $\beta>0$ konstanta.

Rj. U zadatku ćemo iskoristiti osobinu linearnosti Laplasove transformacije

$$\mathcal{L}\{c_1 f_1 + c_2 f_2\} = c_1 \mathcal{L}\{f_1\} + c_2 \mathcal{L}\{f_2\}$$

c_1, c_2 su konstante

$$\operatorname{sh} \beta t = \frac{e^{\beta t} - e^{-\beta t}}{2} = \frac{1}{2}\left(e^{\beta t} - e^{-\beta t}\right)$$

U jednom od prethodnih zadataka smo izračunali da je

$$\mathcal{L}\{e^{\alpha t}\}(s) = \frac{1}{s-\alpha}, \quad s>\alpha$$

Sad imamo

$$\mathcal{L}\{\operatorname{sh} \beta t\}(s) = \mathcal{L}\left\{\frac{1}{2}\left(e^{\beta t} - e^{-\beta t}\right)\right\}(s) =$$

$$= \frac{1}{2}\mathcal{L}\{e^{\beta t}\} - \frac{1}{2}\mathcal{L}\{e^{-\beta t}\}(s) =$$

$$= \frac{1}{2}\cdot\frac{1}{s-\beta} - \frac{1}{2}\cdot\frac{1}{s+\beta} = \frac{1}{2}\cdot\frac{s+\beta-(s-\beta)}{(s-\beta)(s+\beta)} = \frac{\beta}{s^2-\beta^2},$$

za $s>\beta$

Prema tome

$$\mathcal{L}\{\operatorname{sh} \beta t\}(s) = \frac{\beta}{s^2-\beta^2} \quad \text{za } s>\beta>0$$

(#) Odrediti Laplasovu transformaciju f-je $f(t) = \cos\alpha t$ gdje je $\alpha > 0$ konstanta.

Rj. U zadatku ćemo iskoristiti osobinu linearnosti Laplasove transformacije

$$\mathcal{L}\{\alpha f + \beta g\} = \alpha \mathcal{L}\{f\} + \beta \mathcal{L}\{g\}$$

α, β su konstante

Prisjetimo se

$$\begin{array}{rl} & e^{i\alpha x} = \cos\alpha x + i \sin\alpha x \\ + & e^{-i\alpha x} = \cos\alpha x - i\sin\alpha x \\ \hline & e^{i\alpha x} + e^{-i\alpha x} = 2\cos\alpha x \end{array}$$

$$\cos\alpha x = \frac{1}{2}\left(e^{i\alpha x} + e^{-i\alpha x}\right)$$

U jednom od prethodnih zadataka smo pokazali da je

$$\mathcal{L}\{e^{\alpha t}\}(s) = \frac{1}{s-\alpha}, \quad s > \alpha$$

pa je $\mathcal{L}\{e^{(i\alpha)x}\}(s) = \dfrac{1}{s-i\alpha}$ iz čega slijedi da

$$\mathcal{L}\{\cos\alpha t\}(s) = \mathcal{L}\left\{\frac{1}{2}\left(e^{i\alpha x} + e^{-i\alpha x}\right)\right\} = \frac{1}{2}\mathcal{L}\{e^{i\alpha x}\}(s) + \frac{1}{2}\mathcal{L}\{e^{-i\alpha x}\}$$

$$= \frac{1}{2}\cdot\frac{1}{s-i\alpha} + \frac{1}{2}\frac{1}{s+i\alpha} = \frac{1}{2}\cdot\frac{s+i\alpha+s-i\alpha}{(s-i\alpha)(s+i\alpha)} = \frac{s}{s^2+\alpha^2}$$

Prema tome

$$\mathcal{L}\{\cos\alpha t\}(s) = \frac{s}{s^2+\alpha^2} \quad \text{za } \alpha > 0$$

Odrediti Laplace-ovu transformaciju f-je

$$f(t) = \sin 2t \cdot \cos 5t.$$

Rj.

$$
\begin{aligned}
&\sin(A+B) = \sin A \cos B + \sin B \cos A \\
+\ &\sin(A-B) = \sin A \cos B - \sin B \cos A \\
\hline
&\sin(A+B) + \sin(A-B) = 2 \sin A \cos B
\end{aligned}
$$

$$\sin A \cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$$

Prema tome

$$\sin 2t \cos 5t = \frac{1}{2}\left(\sin 7t + \sin(-3t)\right) =$$

$$= \frac{1}{2} \sin 7t - \frac{1}{2} \sin 3t.$$

Znamo da je $\mathcal{L}\{\sin at\}(s) = \dfrac{a}{s^2+a^2}$, $s>0$

Prema tome

$$\mathcal{L}\{\sin 2t \cdot \cos 5t\}(s) = \frac{1}{2}\mathcal{L}\{\sin 7t\}(s) - \frac{1}{2}\mathcal{L}\{\sin 3t\}(s)$$

$$= \frac{1}{2} \cdot \frac{7}{s^2+7^2} - \frac{1}{2} \cdot \frac{3}{s^2+3^2}$$

Prema tome

$$\mathcal{L}\{\sin 2t \cdot \cos 5t\}(s) = \frac{1}{2}\left(\frac{7}{s^2+49} - \frac{3}{s^2+9}\right)$$

Neka je $F(s)=\mathcal{L}\{f\}(s)$. Odrediti $\mathcal{L}\{f(at)\}(s)$, gdje je $a>0$.

Rj.

$$\mathcal{L}\{f(at)\}(s)=\int_0^\infty e^{-st}f(at)\,dt=\left|\begin{array}{ll} at=u & t=\frac{u}{a} \\ a\,dt=du & \\ dt=\frac{1}{a}du & t\big|_0^\infty \Rightarrow u\big|_0^\infty \end{array}\right|$$

$$=\frac{1}{a}\int_0^\infty e^{-\frac{s}{a}u}f(u)\,du=\frac{1}{a}F\left(\frac{s}{a}\right)$$

Prema tome

$$\mathcal{L}\{f(at)\}(s)=\frac{1}{a}F\left(\frac{s}{a}\right)$$

Ako je $F(s)=\mathcal{L}\{f(t)\}(s)$ i $b>0$ odrediti $\mathcal{L}\left\{\frac{1}{b}f\left(\frac{t}{b}\right)\right\}(s)$.

Rj.

$$\mathcal{L}\left\{\frac{1}{b}f\left(\frac{t}{b}\right)\right\}(s)=\frac{1}{b}\mathcal{L}\left\{f\left(\frac{t}{b}\right)\right\}(s)=\frac{1}{b}\int_0^\infty e^{-st}f\left(\frac{t}{b}\right)dt=$$

$$=\left|\begin{array}{ll} \frac{t}{b}=u & t\big|_0^\infty \Rightarrow u\big|_0^\infty \\ \frac{1}{b}dt=du & t=bu \\ dt=b\,du & \end{array}\right|=\int_0^\infty e^{-(bs)u}f(u)\,du=F(bs)$$

Prema tome

$$F(bs)=\mathcal{L}\left\{\frac{1}{b}f\left(\frac{t}{b}\right)\right\}(s)$$

Ako je $F(s) = \mathcal{L}\{f(t)\}(s)$ odrediti $\mathcal{L}\{e^{-at} f(t)\}(s)$.

Rj.: $$\mathcal{L}\{\underbrace{e^{-at} f(t)}_{g(t)}\}(s) = \int_0^{\infty} e^{-st} \underbrace{e^{-at} f(t)}_{g(t)}\, dt =$$

$$= \int_0^{\infty} e^{-(s+a)t} f(t)\, dt = F(s+a).$$

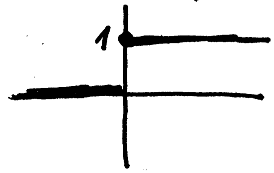
Prema tome $\mathcal{L}\{e^{-at} f(t)\}(s) = F(s+a)$

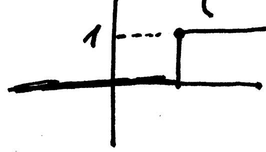
Ako je $\mathcal{L}\{f(t)\}(s) = F(s)$, $a>0$ i $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t<0 \end{cases}$

odrediti $\mathcal{L}\{f(t-a)u(t-a)\}(s)$.

Rj.: $$\mathcal{L}\{f(t-a)u(t-a)\}(s) = \int_0^{\infty} e^{-st} f(t-a)u(t-a)\, dt =$$

$$= \left| u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t<0 \end{cases} \Rightarrow u(t-a) = \begin{cases} 1, & t-a \geq 0 \\ 0, & t-a<0 \end{cases} \right| = \int_a^{\infty} e^{-st} f(t-a)\, dt =$$





$$= \left| \begin{matrix} u = t-a & e^{-st} = e^{-s(u+a)} = e^{-sa} e^{-su} \\ du = dt & \\ t = u+a & t\Big|_a^{\infty} \Rightarrow u\Big|_0^{\infty} \end{matrix} \right| = e^{-sa} \int_0^{\infty} e^{-su} f(u)\, du = e^{-as} F(s)$$

Zadaci za vježbu

1. Koristeći direktno definiciju transformata, odrediti Laplasovu transformaciju sljedećih f-ja

(a) t (b) t^2 (c) e^{6t} (d) te^{3t}

(e) $\cos 2t$ (f) $\cos bt$, b-konstanta (g) $e^{2t}\cos 3t$

(h) $e^{-t}\sin 2t$ (i) $f(t)=\begin{cases} 0, & 0<t<2 \\ t, & 2<t \end{cases}$

(j) $f(t)=\begin{cases} 1-t, & 0<t<1 \\ 0, & 1<t \end{cases}$ (k) $f(t)=\begin{cases} \sin t, & 0<t<\pi \\ 0, & \pi<t \end{cases}$

(l) $f(t)=\begin{cases} e^{2t}, & 0<t<3 \\ 1, & 3<t \end{cases}$

2. Koristeći linearnost Laplasove transformacije i tablicu elementarnih transformata, odrediti slike sljedećih f-ja

(a) t^3-3t^2+2 (b) $(t+1)^3$ (c) $3e^t \operatorname{sh} 2t$

(d) $\operatorname{ch} 2t \cdot \operatorname{sh} t$ (e) $\cos^2(2t)$ (f) $\sin t \cdot \sin 2t$

(g) $\cos 5t \cdot \sin 3t$ (h) $\cos^3 t$

2. (g) $\dfrac{4}{s^2+64}-\dfrac{3}{s^2+36}$

Odabrana rješenja:

1. (a) $\frac{1}{s^2}$ (c) $\dfrac{1}{s-6}$, $s>6$ (e) $\dfrac{s}{s^2+4}$, $s>0$ (g) $\dfrac{s-2}{(s-2)^2+9}$, $s>2$

(i) $e^{-2s}\left(\dfrac{2s+1}{s^2}\right)$, $s>0$ (k) $\dfrac{e^{-\pi s}+1}{s^2+1}$, $\forall s$

2. (f) $\frac{1}{2}\left(\dfrac{s}{s^2+1}-\dfrac{s}{s^2+9}\right)$

Kraća tabela Laplasovih transformacija

$f(t)$	$F(s)=\mathcal{L}\{f\}(s)$
1	$\frac{1}{s}, \quad s>0$
$e^{\alpha t}$	$\frac{1}{s-\alpha}, \quad s>\alpha$
$t^n, \; n=1,2,\dots$	$\frac{n!}{s^{n+1}}, \quad s>0$
$\sin\beta t$	$\frac{\beta}{s^2+\beta^2}, \quad s>0$
$\cos\beta t$	$\frac{s}{s^2+\beta^2}, \quad s>0$
$e^{\alpha t}t^n, \; n=1,2,\dots$	$\frac{n!}{(s-\alpha)^{n+1}}, \quad s>\alpha$
$e^{\alpha t}\sin\beta t$	$\frac{\beta}{(s-\alpha)^2+\beta^2}, \quad s>\alpha$
$e^{\alpha t}\cos\beta t$	$\frac{s-\alpha}{(s-\alpha)^2+\beta^2}, \quad s>\alpha$
t	$\frac{1}{s^2}, \quad s>0$
$\mathrm{ch}\,\alpha t$	$\frac{s}{s^2-\alpha^2}, \quad s>\lvert\alpha\rvert$
$\mathrm{sh}\,\alpha t$	$\frac{\alpha}{s^2-\alpha^2}, \quad s>\lvert\alpha\rvert$